

Laminar Micropolar Nanofluid Flow in a Porous Medium with Variable Permeability Considering Heat Source/Sink

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-----ABSTRACT-----

An analysis is conducted for laminar micropolar nanofluid flow in an upright conduit filled with a porous medium. The permeability of the porous substrate is presumed to be varying across the width of the channel while a heat source/sink is also pondered. The governing equations are solved using the homotopy analysis method. Graphs are presented for the effects of permeability parameter and micropolar parameter on velocity, microrotation and temperature for both heat source and heat sink. It was perceived that increasing the permeability intensified the fluid flow velocity while the microrotation decreased for the same. The velocity, microrotation and temperature of the fluid was all observed to be lower for heat sink than for heat source.

KEYWORDS: -Micropolar, Nanofluid, Variable Permeability, Heat Source/Sink

I. INTRODUCTION

Nanofluids are fluids containing 1-100 nm of suspended nanoparticles. Usually the base fluids used are water, mineral oil, ethylene glycol, etc., while usually the nanoparticles used are copper, silver, alumina, and titanium. Since Choi[1] coined the term 'Nanofluids', the field has advanced rapidly in the proceeding years. Both academics and industrialists have taken a keen interest in the theory and application development of the field. It has been found that nanofluid increases the fluid heat transfer capacity. In the following articles, general applications of nanofluids were reviewed [2] and [3].

Eringen[4] introduced the theory of micropolar fluids considering fluids with micro-rotational effects and microrotational inertia. Micropolar fluids are fluids with their own spins and rotations composed of solid, randomly oriented, or spherical particles. Some examples of fluids in this group are animal blood, ferrofluids, liquid crystals. Chamkha et.al[5] considered the free convective flow in a vertical conduit and concluded that an increase in the wall temperature ratio raises, respectively, temperature and speed profiles. Chamkha et. al[6] concluded that the temperature and velocity profiles can be highly skewed in developing regions when mixed micropolar fluid flows in an upright channel. In an upright conduit with unequal temperature of the wall and unequal concentrations, Cheng [7] obtained analytical results and studied the effects of the vortex viscosity parameter and the buoyancy ratio, while Bataineh et.al[8] deliberated the same problem using HAM. Abdulaziz et.al [9] also used HAM to research heat transfer in a vertical channel of micropolar fluid flow and showed that magnetic field and micropolar effects are useful in minimizing heating effects and in regulating the heat transfer rate. Kumar et.al[10] demonstrated that micropolar fluids act as cooling agents as well as reducing drags on the flow surface walls. In view of the effects of slip conditions, Ashmawy[11] researched a fully developed natural convective flow.

Micropolar nanofluids are micropolar fluids that have additional nanoparticles. In a porous channel with radiation effects, Mohyud-Din et.al.[12] deliberated magnetohydrodynamic(MHD) flow. Hussain et.al[13] concluded that a change in the volume fraction of nanoparticles causes Nusselt number to decline while increasing the profiles of velocity and temperature, respectively. Turk and Tezer-Sezgin[14] were the first to use FEM to solve micropolar nanofluid flow equations that are highly nonlinear. Noor et al.[15] studied the stagnation flow of micropolar nanofluids and the effects of slip on the heat transfer rate. For a more comprehensive study of the micropolar fluid, the readers may read the following articles [16]–[21]

Flow in porous medium is a subject of intense research and study as it has several practical applications and also, they are found in many natural flows in nature. It has been noted that the permeability of a porous medium may not always be a constant. Pioneering work for flow in a channel filled with a porous medium with variable permeability was investigated by Chandrasekhara [22] and Chandrasekhara and Namboodri[23]. Exponential variation of permeability of the porous medium was taken into consideration by Ibrahim and Hassanien [24]. Zaytoon[25] then proposed a quadratic variation in permeability in his study of flow in a variable porous medium.

The literature on micropolar nanofluids are still very limited. The presented investigation studies the mixed convection micropolar nanofluid flowing in an upright channel filled with an exponentially varying

porous medium while considering the influence of heat source/sink. The micropolar nanofluid mathematical model is a coupled differential equation which are then solved using HAM proposed by Liao[26]. The influence of significant parameters on the flow are reported graphically and deliberated.

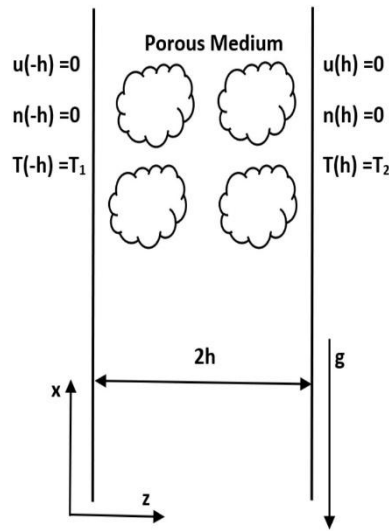


Fig. 1 Schematic diagram

II. PROBLEM DESCRIPTION

In an upright channel filled by a porous medium having variable permeability, a laminar micropolar nanofluid flow is considered. Copper nanoparticles are assumed to be used for the nanoparticles while heat source and sink is also considered to be present in the flow. The walls are postulated to be kept at a distance $2h$ apart as displayed in Fig. 1. The physical properties of the micropolar fluid are taken to be constant while Boussinesq approximation is also taken into consideration. Taking the above assumptions in place, the equations governing the flow can be expressed as[27], [28]:

$$(\mu_{nf} + \kappa) \frac{d^2 u}{dz^2} + \kappa \frac{dn}{dz} + g(\rho\beta)_{nf} (T - T_1) - \frac{\mu_{nf}}{K(z)} u = \frac{dp}{dx} \quad (1)$$

$$\gamma_{nf} \frac{d^2 n}{dz^2} - \kappa \left[2n + \frac{du}{dz} \right] = 0 \quad (2)$$

$$k_{nf} \frac{d^2 T}{dz^2} + Q(T - T_1) = 0 \quad (3)$$

Subject to the conditions at the boundary as:

$$u(-h) = 0, n(-h) = 0, T(-h) = T_1 \quad (4a)$$

$$u(h) = 0, n(h) = 0, T(h) = T_2 \quad (4b)$$

where subscripts nf, s denotes the nanofluid property, micropolar fluid property and nanoparticle property, respectively, u indicates the velocity component on the vertical channel, T denotes the temperature of the fluid, n denotes the angular velocity of the micropolar fluid, Q denote the heat source/sink, φ denotes the volume fraction of the nanoparticle, j denotes the microinertia density, κ denotes the viscosity of the vortex, $\gamma_{nf} = (\mu_{nf} + \frac{\kappa}{2})j$ denotes the spin gradient viscosity, μ denotes the dynamic viscosity of the fluid, ρ_{nf} denotes the nanofluid density, g the acceleration due to gravity, β_{nf} denotes the coefficient of thermal expansion. The permeability of the porous medium is taken as $K(z) = K_0 e^{\delta z}$ [29], varying exponentially across the width of the channel, where δ is the permeability parameter.

Introduction of variables for non-dimensional transformation

$$X = \frac{x}{h}, Z = \frac{z}{h}, u = \frac{U}{U_0}, \theta = \frac{T - T_1}{T_2 - T_1}, N = h \frac{n}{U_0} \quad (5)$$

into Eqns. (1) – (3), we get the dimensionless equations as presented below:

$$[A_1 + c] \frac{d^2 U}{dZ^2} + c \frac{dN}{dZ} + A_2 \frac{Gr}{Re} \theta - ReP - A_1 \frac{U}{Da e^{dZ}} = 0 \quad (6)$$

$$[A_1 + \frac{c}{2}] \frac{d^2 N}{dZ^2} - c \left(2N + \frac{dU}{dZ} \right) = 0 \quad (7)$$

$$A_3 \frac{d^2 \theta}{dZ^2} + \psi \theta = 0 \quad (8)$$

where $c = \frac{\kappa}{\mu_f}$ is micropolarity parameter, $Da = \frac{K_0}{h^2}$ is Darcy number, $Gr = \frac{\rho_f^2 \beta_f g h^3 (T_2 - T_1)}{\mu_f^2}$ is Grashof number, $Re = \frac{\rho_f U_0 h}{\mu_f}$ is Reynolds number, $P = \frac{d\bar{P}}{dx}$ is dimensionless pressure gradient, $d = \frac{\delta}{h}$ is the permeability parameter, ψ is the heat source/sink parameter, $A_1 = \frac{1}{(1-\phi)^{2.5}}$, $A_2 = \left[(1-\phi) + \phi \frac{(\rho\beta)_s}{(\rho\beta)_f} \right]$ and $A_3 = \frac{k_{nf}}{k_f}$.

The dimensionless boundary conditions are

$$U(-1) = 0, \theta(-1) = 0, N(-1) = 0 \tag{9a}$$

$$U(1) = 0, \theta(1) = 1, N(1) = 0 \tag{9b}$$

III. SOLUTION METHOD

The dimensionless equations presented in Eqn. (6)-(8) along with the conditions Eqn. (9) are solved using the homotopy analysis method [26]. As a requirement we define the linear operators as:

$$L_i = \frac{d^2}{dz^2}, i=1,2,3 \tag{10}$$

such that $L_1(C_1 + C_2Z) = 0, L_2(C_3 + C_4Z) = 0, L_3(C_5 + C_6Z) = 0$ and $C_i, i = 1 - 6$ are constants. With reference to the linear operator and the boundary condition, the guess approximations are defined as below:

$$U_0 = 0, N_0 = 0, \theta_0 = 0 \tag{11}$$

The m^{th} - order deformation equations are defined as:

$$L_1[U_m(Z) - \chi_m U_{m-1}(Z)] = h_1 G_m^U(Z) \tag{12}$$

$$L_2[N_m(Z) - \chi_m N_{m-1}(Z)] = h_2 G_m^N(Z) \tag{13}$$

$$L_3[\theta_m(Z) - \chi_m \theta_{m-1}(Z)] = h_3 G_m^\theta(Z) \tag{14}$$

with the boundary conditions:

$$U(0;p)_{m-1} = 0; N(0;p)_{m-1} = 0; \theta(0;p)_{m-1} = 0 \tag{15}$$

$$U(1;p)_{m-1} = 0; N(1;p)_{m-1} = 0; \theta(1;p)_{m-1} = 0 \tag{16}$$

where

$$G_m^U(Z) = [A_1 + c] \frac{d^2 U_{m-1}}{dz^2} + c \frac{dN_{m-1}}{dz} + A_2 \frac{Gr}{Re} \theta_{m-1} - ReP - A_1 \frac{U_{m-1}}{Da e^{dz}}$$

$$G_m^N(Z) = \left[A_1 + \frac{c}{2} \right] \frac{d^2 N_{m-1}}{dz^2} - c \left(2N_{m-1} + \frac{dU_{m-1}}{dz} \right)$$

$$G_m^\theta(Z) = A_3 \frac{d^2 \theta_{m-1}}{dz^2} + \psi \theta_{m-1}$$

where m is an integer and

$$\chi_m = \begin{cases} 0 & \text{for } m \leq 1 \\ 1 & \text{for } m > 1 \end{cases}$$

p is the embedding parameter and h_1, h_2, h_3 are control the convergence of the solution. Using the Taylor's series expansion, the approximation in Eqn. (11) and putting $p=1$, the solution expression is presented as:

$$U(Z) = U_0(Z) + \sum_{m=1}^{\infty} U_m(Z)$$

$$N(Z) = N_0(Z) + \sum_{m=1}^{\infty} N_m(Z)$$

$$\theta(Z) = \theta_0(Z) + \sum_{m=1}^{\infty} \theta_m(Z)$$

such that

$$U_m(Z) = \frac{1}{m!} \left. \frac{\partial^m U(Z;p)}{\partial p^m} \right|_{p=1}$$

$$N_m(Z) = \frac{1}{m!} \left. \frac{\partial^m N(Z;p)}{\partial p^m} \right|_{p=1}$$

$$\theta_m(Z) = \frac{1}{m!} \left. \frac{\partial^m \theta(Z;p)}{\partial p^m} \right|_{p=1}$$

The convergence of the HAM solution relies on the h parameter. Hence, it is necessary to find the optimum value. For this process, the average residual error is defined as below[30]:

$$E_{U,m} = \frac{1}{K} \sum_{i=0}^K \left[G_1 \left(\sum_{j=0}^m U_k(j\Delta y) \right) \right]^2$$

$$E_{N,m} = \frac{1}{K} \sum_{i=0}^K \left[G_2 \left(\sum_{j=0}^m N_k(j\Delta y) \right) \right]^2$$

$$E_{\theta,m} = \frac{1}{K} \sum_{i=0}^K \left[G_3 \left(\sum_{j=0}^m \theta_k(j\Delta y) \right) \right]^2$$

where $\Delta y = 1/K$ and $K = 40$. The averaged residual errors with least values are displayed in Table 1 for different orders of approximation. Hence, the h value for U is taken at $h_1 = -0.66$, for N is taken at $h_2 = -0.9$ and for θ is taken as $h_3 = -1.06$

Table 1: Minimum error for various orders of approximation

Order(m)	h_1	Minimum error	h_2	Minimum error	h_3	Minimum error
8	-0.66	1.59×10^{-6}	-0.9	9.97×10^{-9}	-1.06	3.79×10^{-12}
9	-0.66	1.19×10^{-7}	-0.9	1.87×10^{-9}	-1.06	1.74×10^{-13}
10	-0.66	6.57×10^{-9}	-0.9	4.19×10^{-11}	-1.06	8.09×10^{-15}

Table 2: Comparison between nanofluid and micropolar nanofluid

Parameters		Micropolar Nanofluid		Nanofluid	
		$U(-0.5)$	$U(0.5)$	$U(-0.5)$	$U(0.5)$
$d=-1.5$	$\psi = -1$	4.9547	7.4071	5.0864	7.5912
$d=-2.5$		4.3318	4.9582	4.5995	5.3408
$d=-1.5$	$\psi = 1$	13.9939	15.2166	14.3919	15.5578
$d=-2.5$		13.7085	11.9791	14.1671	12.2662

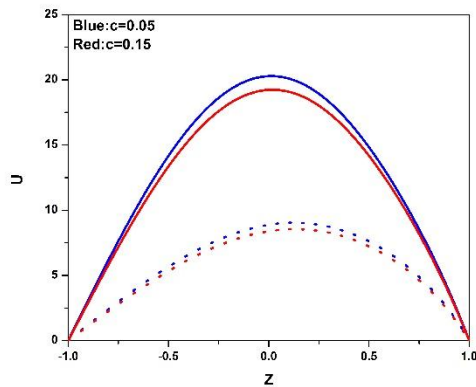


Fig. 2: Change in U with respect to change in c

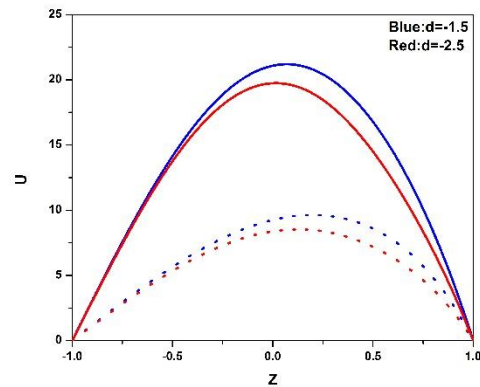


Fig. 3: Change in U with respect to change in d

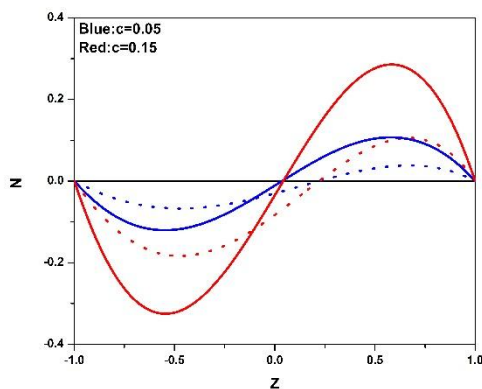


Fig. 4: Change in N with respect to change in c

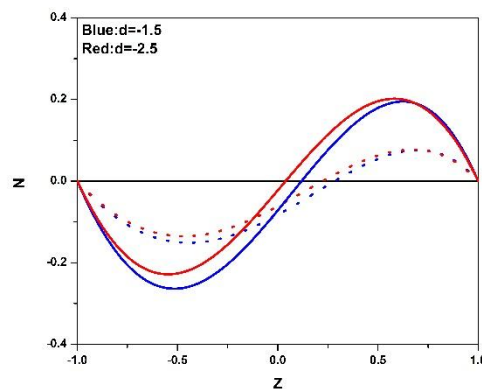


Fig. 5: Change in N with respect to change in d

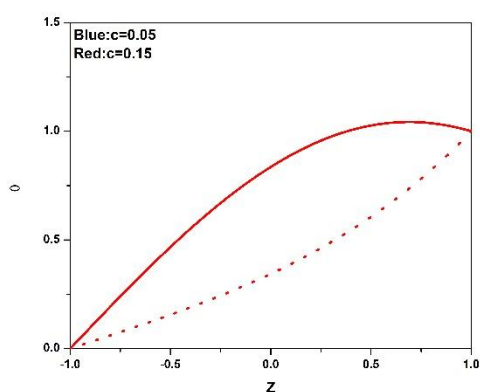


Fig. 6: Change in θ with respect to change in c

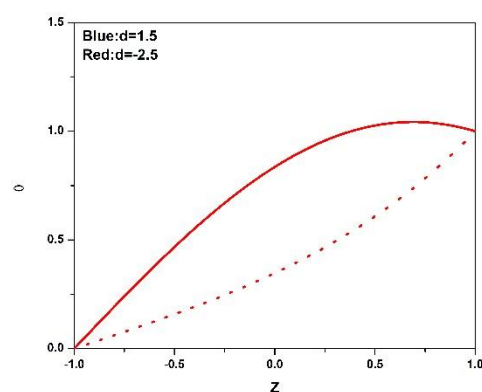


Fig. 7: Change in θ with respect to change in d

IV. RESULTS AND DISCUSSION

In Table 2, we have shown the comparison between a micropolar nanofluid and a normal nanofluid. We can see that the velocity of the nanofluid surpasses the velocity the micropolar nanofluid. This is because the microrotation of the micropolar nanofluid reduces the fluid velocity. Figures (2)-(7) shows the effect of permeability parameter (d) and micropolarity parameter (c) on the flow velocity, microrotation and temperature. It may be noted that the solid lines represent the case of heat source ($\psi = 1$) and dotted lines represent the case of heat sink ($\psi = -1$). Figure 2 shows that increasing c retards the velocity of the flow. This is due to the increase in vortex viscosity when c is increased. Increase in d is detected, from Fig. 3, to increase the velocity. Increasing the permeability allows for more space for the fluid to flow and thus an increase in velocity is observed. Increase in c has a significant impact on microrotation of the fluid, as can be seen in Fig. 4. Increasing c leads to increase in vortex viscosity of the fluid which leads to an augmentation in microrotation of the fluid. Increasing d was observed to bring about a fall in microrotation of the fluid, as displayed in Fig. 5. It can be seen from Figs. 6 and 7 that both c and d do not have any impact on the temperature of the fluid flow. This is due to the independence of the temperature equation (8) from both c and d . It may be observed that for all cases, the heat source ($\psi = 1$) causes a rise in value of the velocity, microrotation and temperature while the opposite phenomena while considering heat sink ($\psi = -1$).

V. CONCLUSION

The paper tackles the problem of micropolar nanofluid flowing in an upright channel packed with a saturated, variable porous medium while taking into consideration heat source/sink. The equations representing the flow were solved by the HAM and graphs were made to discuss the influence of significant parameters on the flow. It was noted that there is a significant difference on the flow regime while taking heat source and sink. The increase in permeability is seen to increase the velocity while retarding the microrotation of the fluid particles.

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